Analytical modeling of oscillatory heat transfer in coated sorption beds

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Outline

- Oscillatory thermal behavior of sorption cooling systems
- Motivation and objective
- Oscillatory heat transfer model
- Model validation
- Parametric study
- Performance evaluation
- Conclusion

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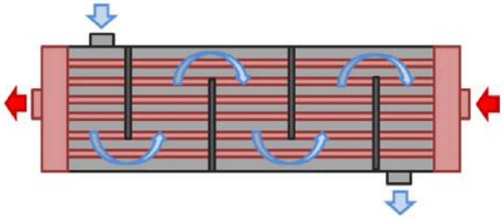
Sorber beds used in sorption cooling systems

Spiral tube HE



Shell and tube HE

Hair pin HE

Plate fin HE

Finned tube HE

Plate HE

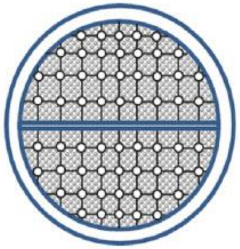
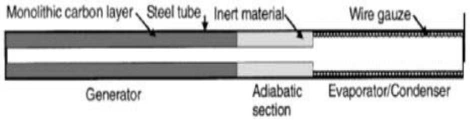


Plate tube HE

Annular tube HE



Simple tube HE

A. Sharafian, M. Bahrami, Assessment of adsorber bed designs in waste-heat driven adsorption cooling systems for vehicle air conditioning and refrigeration, Renew. Sustain. Energy Rev. 30 (2014) 440–451.

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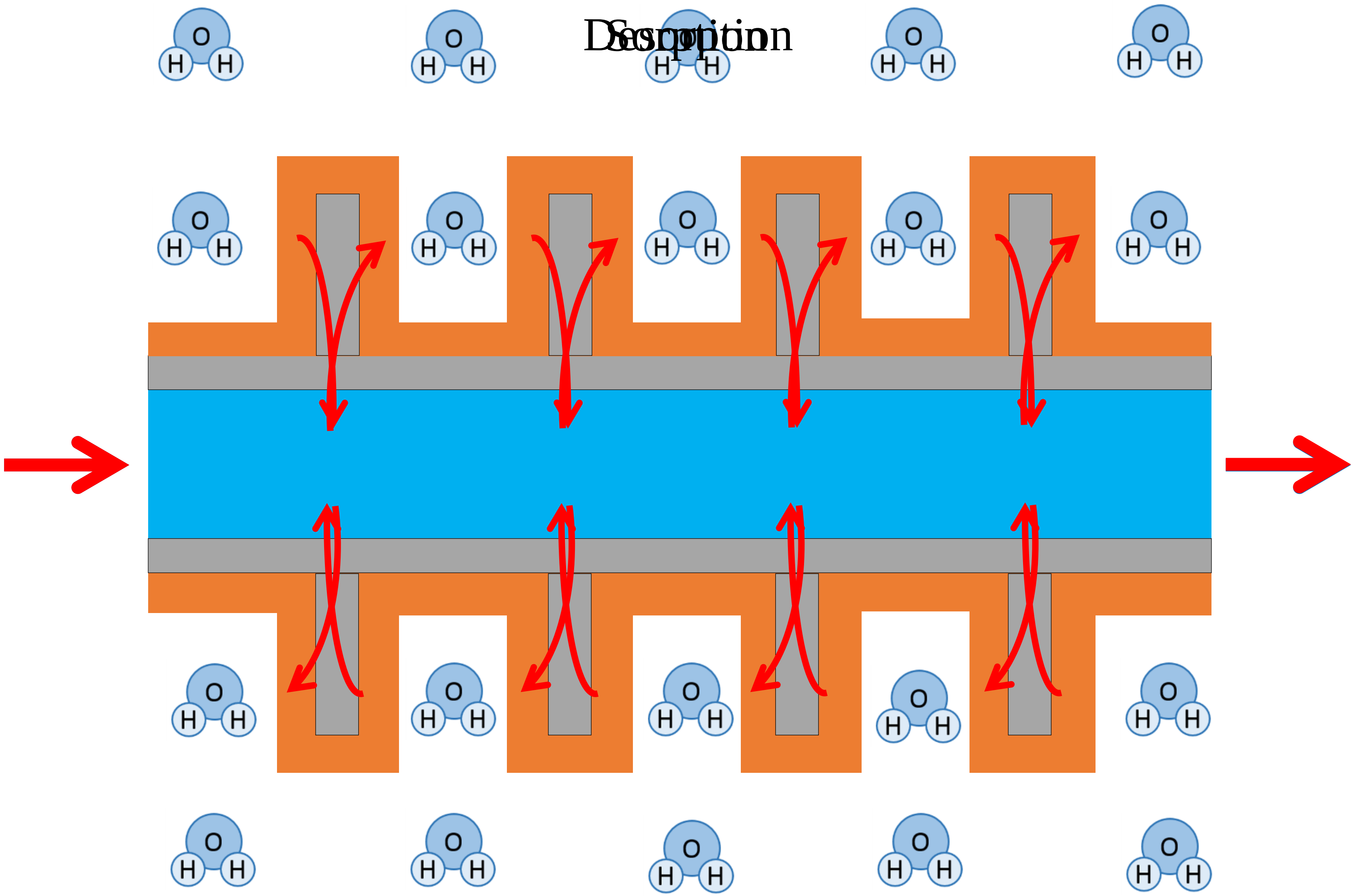
Oscillatory thermal behavior of sorption cooling systems

Desorption
Sorption



Motivation and objective

- Oscillatory heat transfer behavior makes
  - Thermal diffusivity of the heat exchanger (HEX)
  - Thermal diffusivity of the sorbent
  - Thermal contact resistance (TCR) at the sorbent/HEX interface

  crucially important in the performance of a sorption cooling system (SCS)

- Investigation of oscillatory heat transfer characteristics

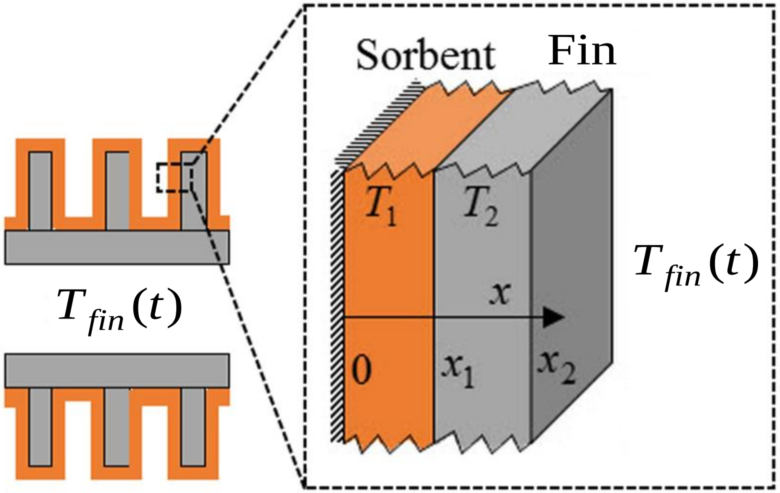
Mathematical modeling of sorption cooling systems

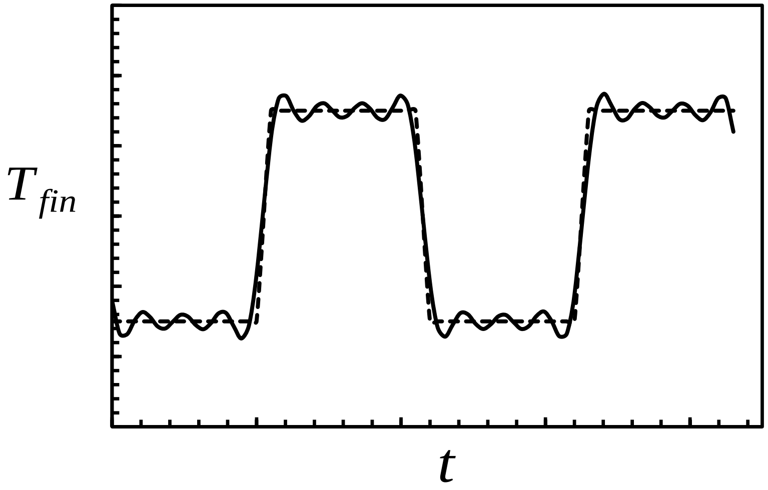
- Thermodynamic modeling (Tamainot-Telto 2009, Henninger 2012)
  - Fairly simple and cost-effective
  - Predict the upper performance limits
- Lumped modeling (Saha 2007, Ahmed 2012)
  - Uniform sorbent temperature
  - Uniform sorption of refrigerant
  - Neglect inter-particle heat and mass transfer resistances
- Heat and mass transfer models (Solmuş 2012, Niazmand 2012)
  - Variation of sorbent temperature and sorbate uptake with time and space
  - Require high computational time
- Analytical model
  - Variation of sorbent temperature and sorbate uptake with time and space
  - Generate closed-form relationships

Modeling and assumptions

➢ Assumptions

- One-dimensional energy equation
- Adiabatic at x=0 due to negligible convection and radiation heat transfer
- Constant thermo-physical properties of the sorbent and the fin
- Negligible convection of the sorbate inside the sorbent coating





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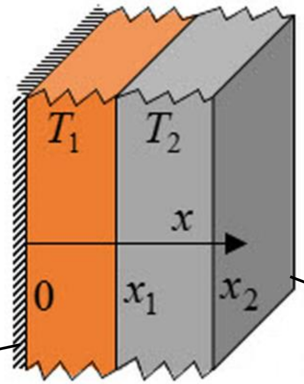
Energy equation in a sorber bed

$$\alpha_i \frac{\partial^2 T_i(x,t)}{\partial x^2} + \frac{\alpha_i}{k_i} G_i(t) = \frac{\partial T_i(x,t)}{\partial t}, \quad x_{i-1} \le x \le x_i, t > 0$$

$$i = 1, 2$$

$$G_i(t) = \begin{cases} \rho_{sorb} H_{sorp} \dfrac{dw}{dt}, & i = 1 \\ 0, & i = 2 \end{cases}$$



$x = 0$

$$\frac{\partial T_1(0,t)}{\partial x} = 0$$

$x = x_1$

$$k_1 \frac{\partial T_1(x_1,t)}{\partial x} = k_2 \frac{\partial T_2(x_1,t)}{\partial x}$$

$$-k_1 \frac{\partial T_1(x_1,t)}{\partial x} = \frac{1}{A \cdot TCR}\left(T_1(x_1,t) - T_2(x_1,t)\right)$$

$x = x_2$

$$T_2(x_2,t) = T_{fin}(t)$$

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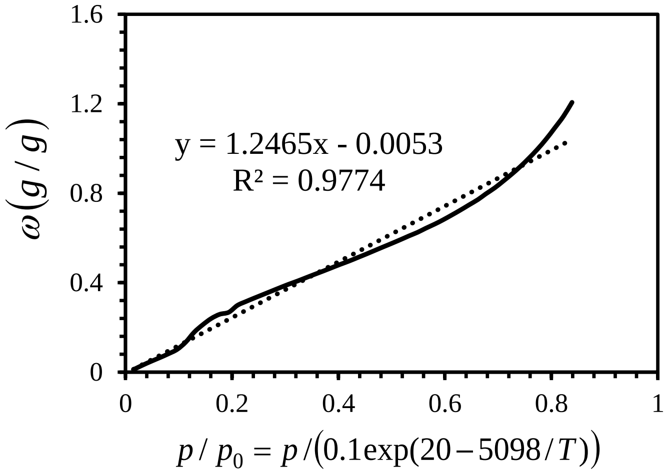
Modeling of sorbate uptake

Objective ⟶ $\omega(T, p)$

- Sorbent material, consisting of 35 wt% $CaCl_2$, 35 wt% silica gel B150, 10 wt% PVP-40, and 20 wt% graphite flakes
- Isotherm plot obtained from IGA-002 thermogravimetric sorption analyzer (TGA)



$$\omega = m\frac{p}{p_0} + b = m\frac{p}{0.1\exp(20-5098/T)} + b \approx m'(p)T + b'(p)$$

$R^2 = 0.9935$

small temperature jump of the sorbent (<15°C)

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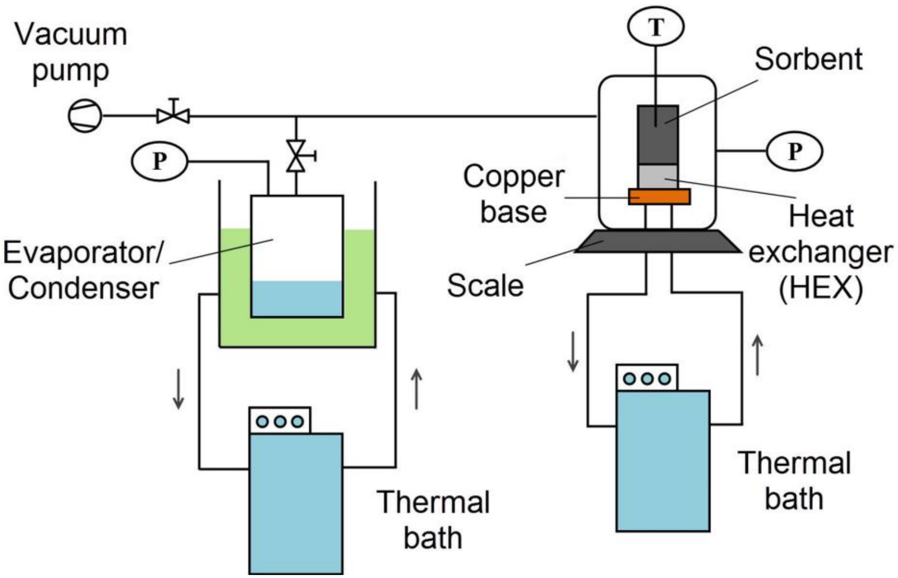
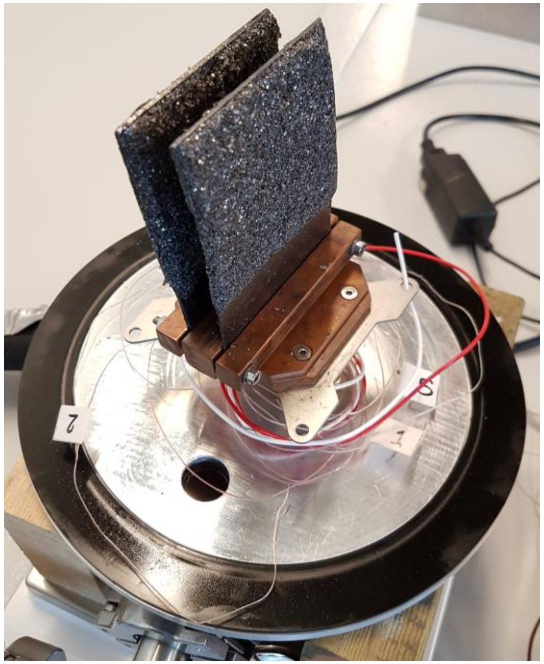
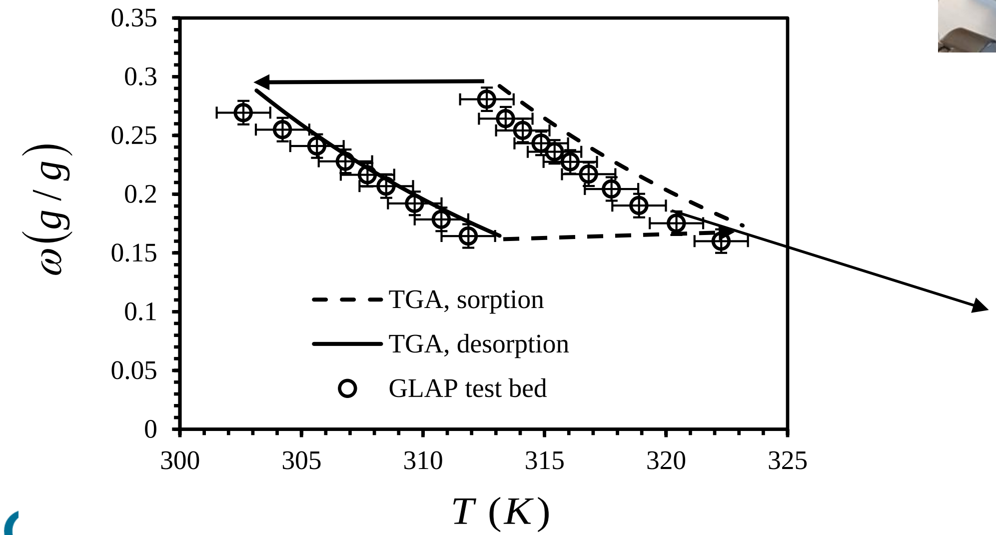
Gravimetric large pressure jump (GLAP) test bed







$$\omega \approx m'(p)T + b'(p)$$

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Solution procedure

Non-dimensionalize the energy equation

Orthogonal Expansion Technique[1]

Establish Eigen-value problem

Obtain transcendental equation

Acquire Eigen-functions

Use orthogonal property of Eigen-functions to form Gamma problem

Solve Gamma problem (ODE)

Obtain compact relationships for temperature distribution

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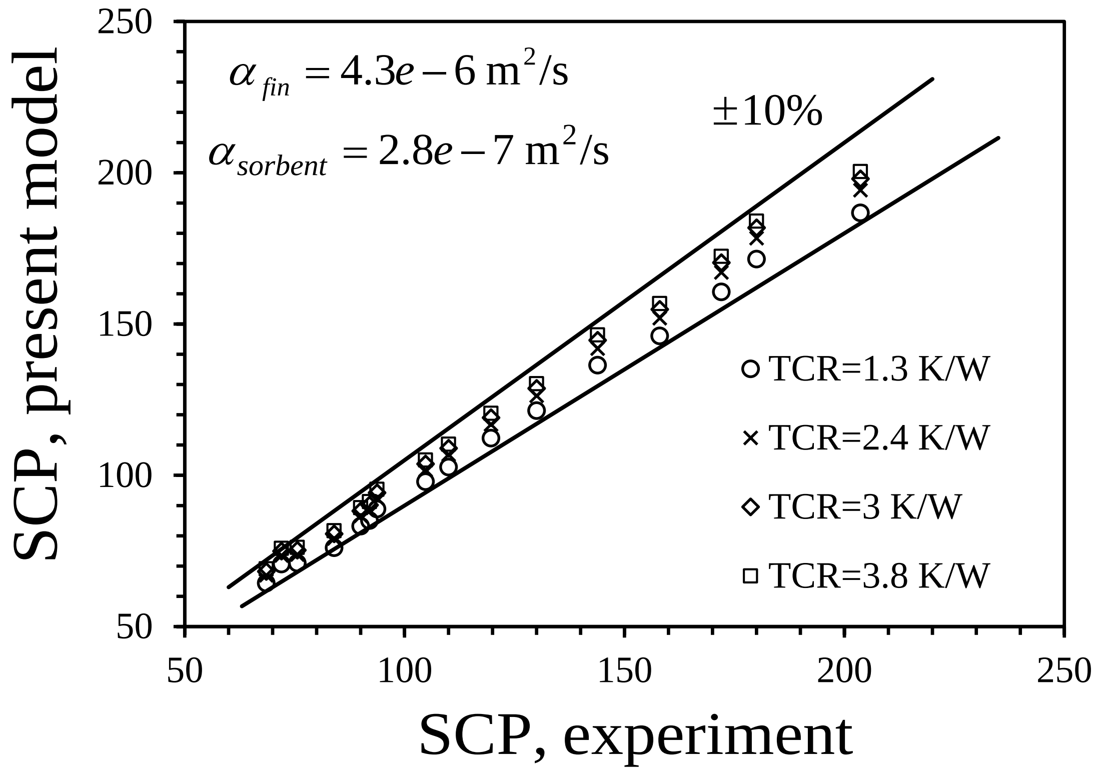
[1] M. Necati Özışık, Boundary value problems of heat conduction, International Textbook Company, 1968

Validation of model with measurements

$$SCP = \frac{Q_{evap}}{m_{sorb}\tau_{cycle}} = \frac{\Delta\omega\, h_{fg@T_{evap}}}{\tau_{cycle}}$$



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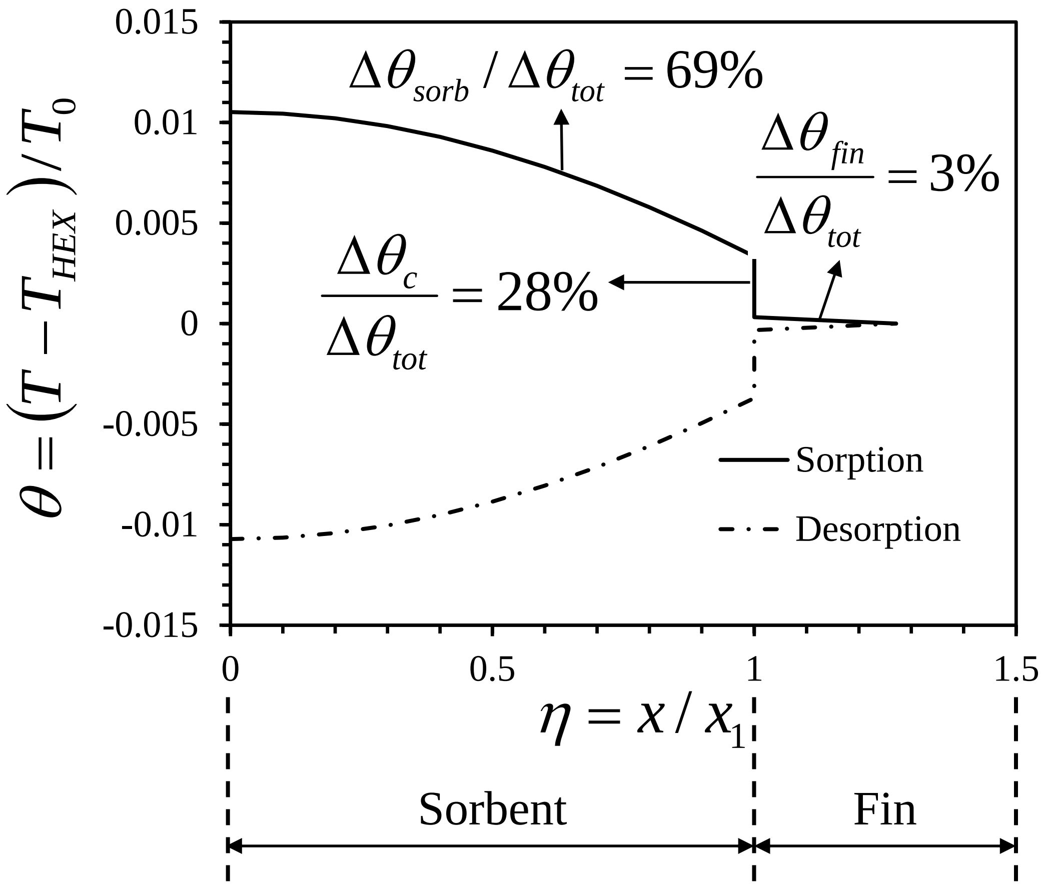
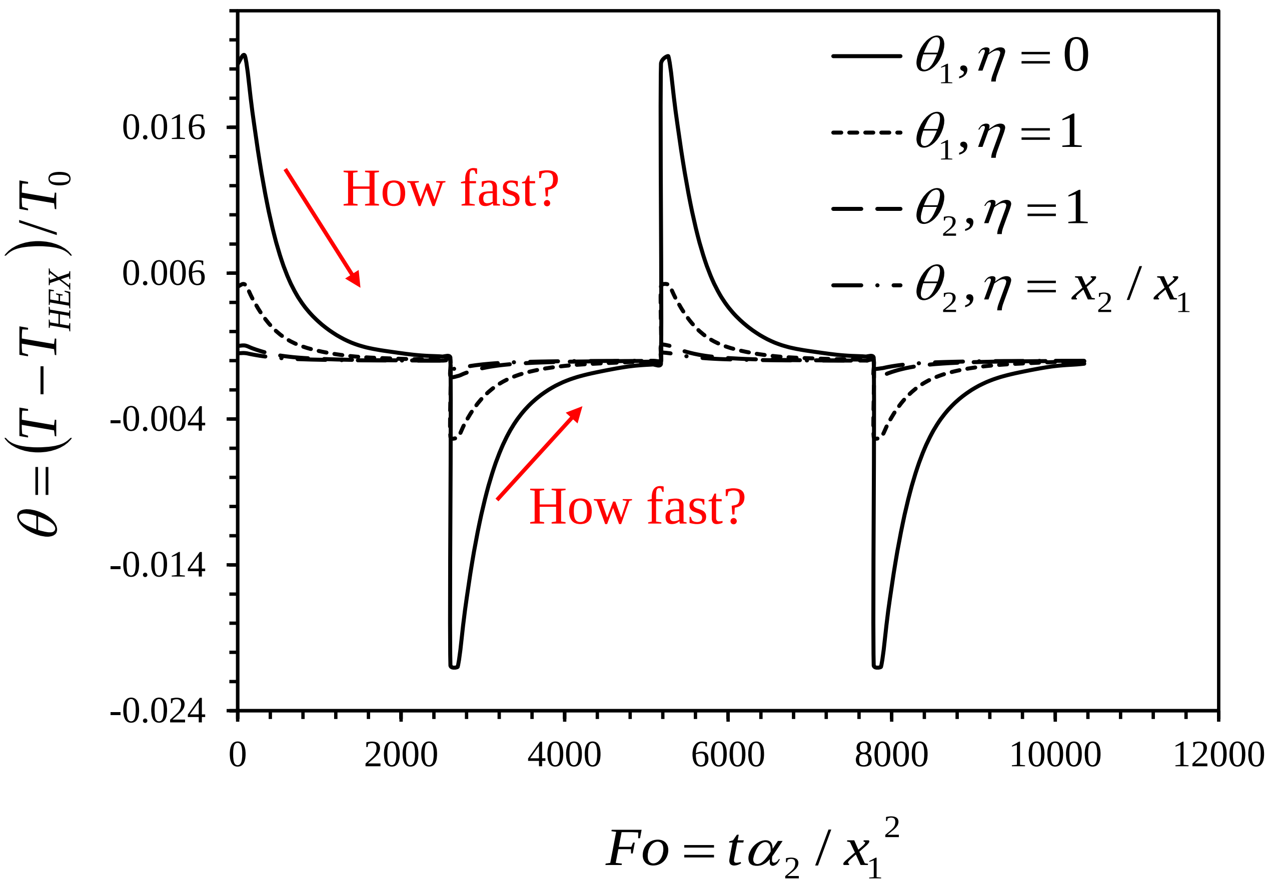
A. Sharafian, K. Fayazmanesh, C. McCague, M. Bahrami, Thermal conductivity and contact resistance of mesoporous silica gel adsorbents bound with polyvinylpyrrolidone in contact with a metallic substrate for adsorption cooling system applications, Int. J. Heat Mass Transfer 79 (2014) 64–71

Temperature distribution inside a sorber bed

Sorbent thermal diffusivity and TCR limit the heat transfer



Oscillatory variation of temperature inside a sorber bed



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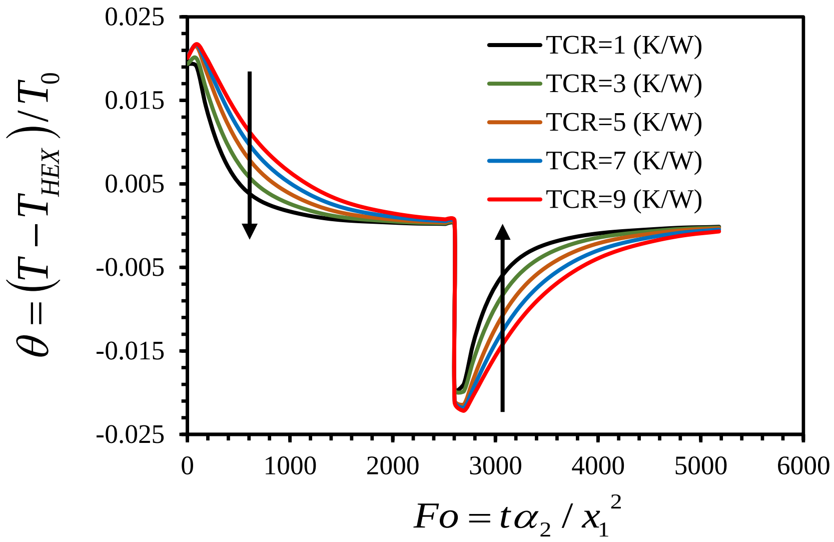
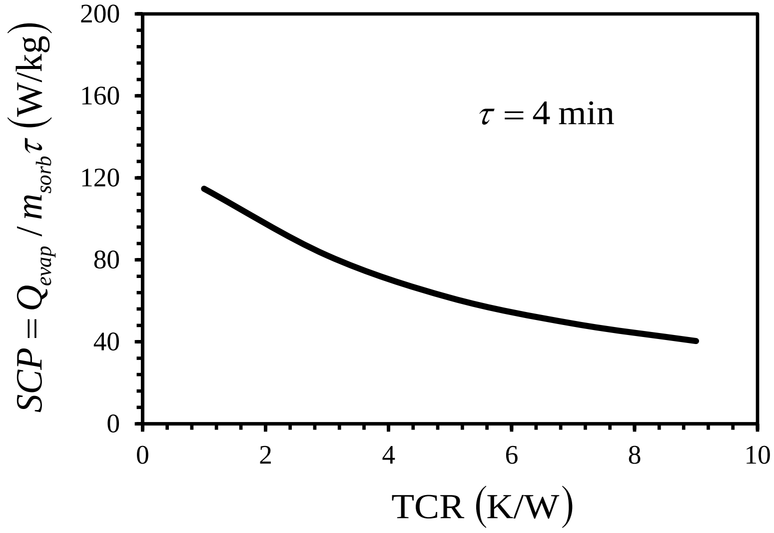
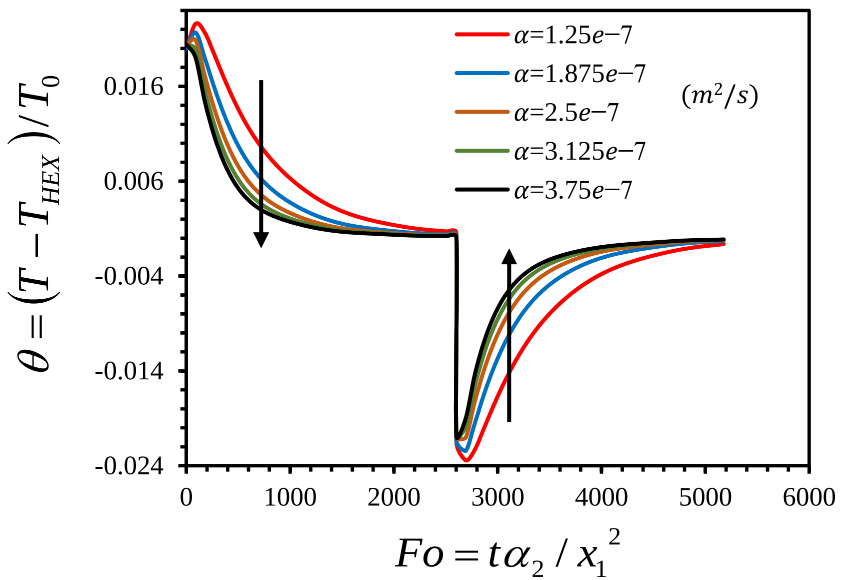
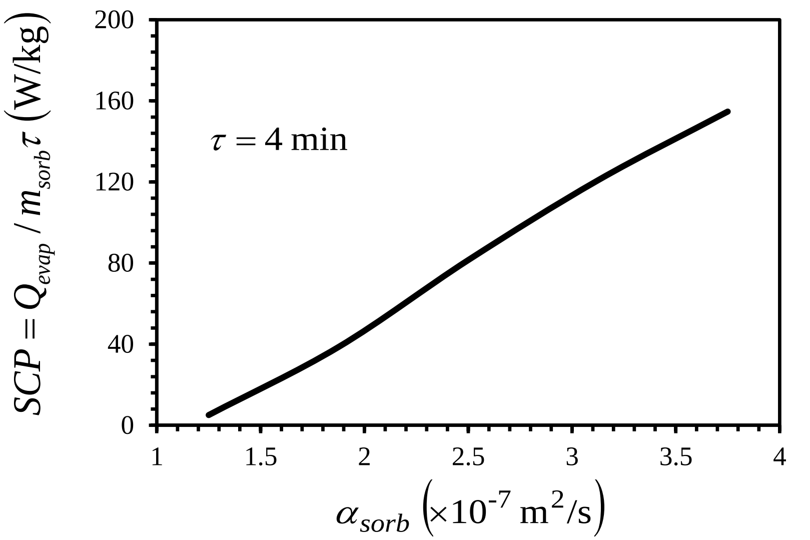
Effect of thermal contact resistance (TCR) on SCP





Effect of sorbent thermal diffusivity on SCP





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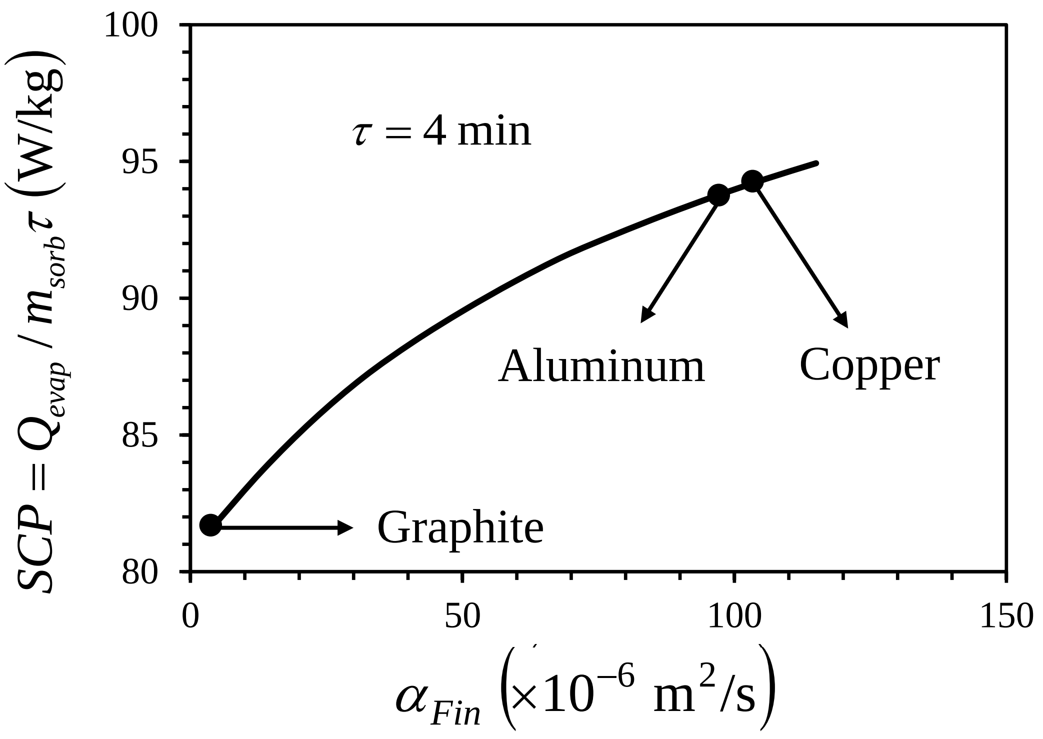
Effect of fin thermal diffusivity on SCP



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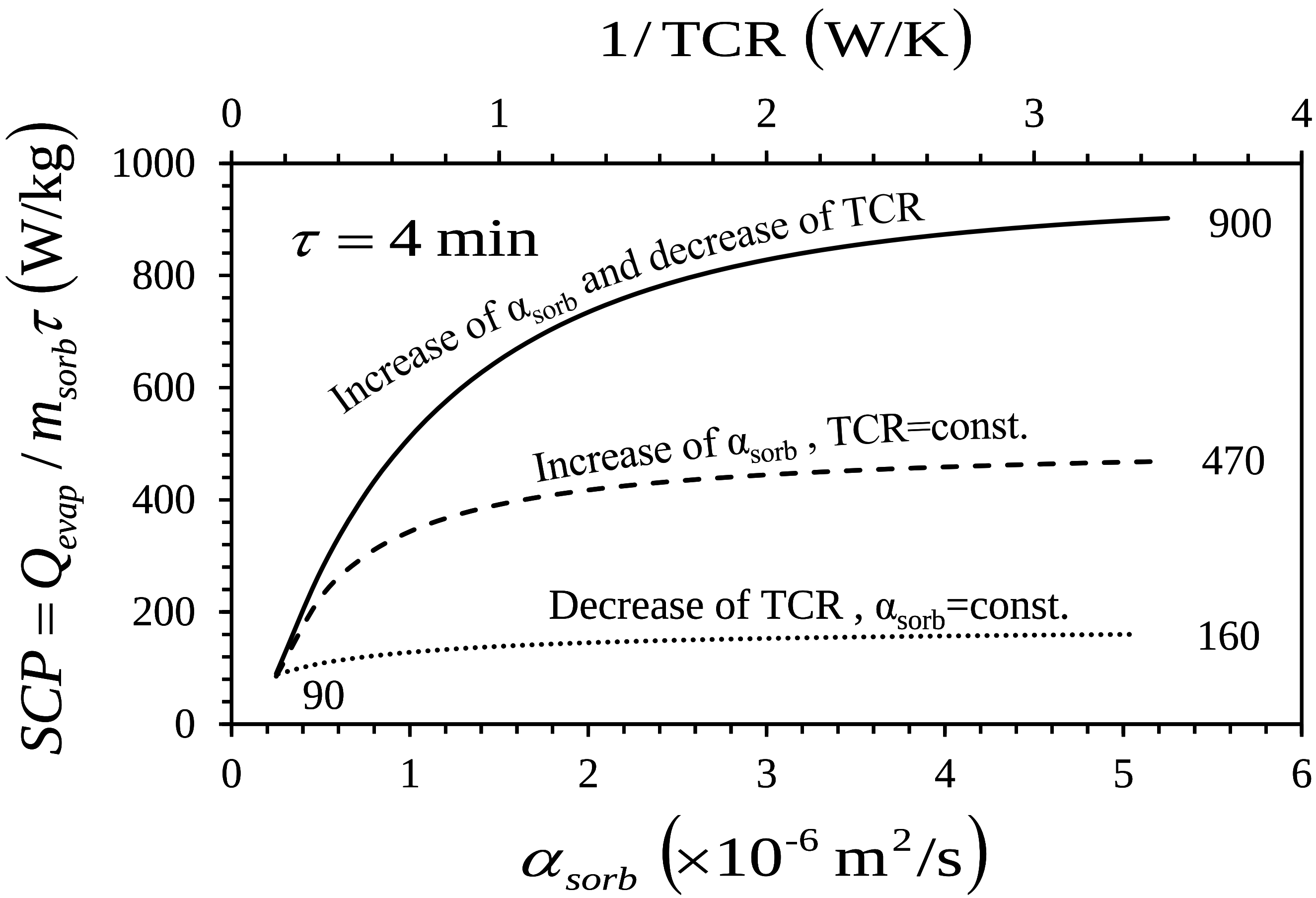
Effect of sorbent thermal diffusivity and TCR on SCP



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Conclusion

- Novel analytical model
  - Oscillatory heat transfer inside sorbent and HEX
  - Including thermal contact resistance
  - Predict performance of a sorption cooling system
- Parametric study and performance evaluation
  - Sorbent thermal diffusivity
  - Thermal contact resistance

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Thank you for your attention

Questions/comments?

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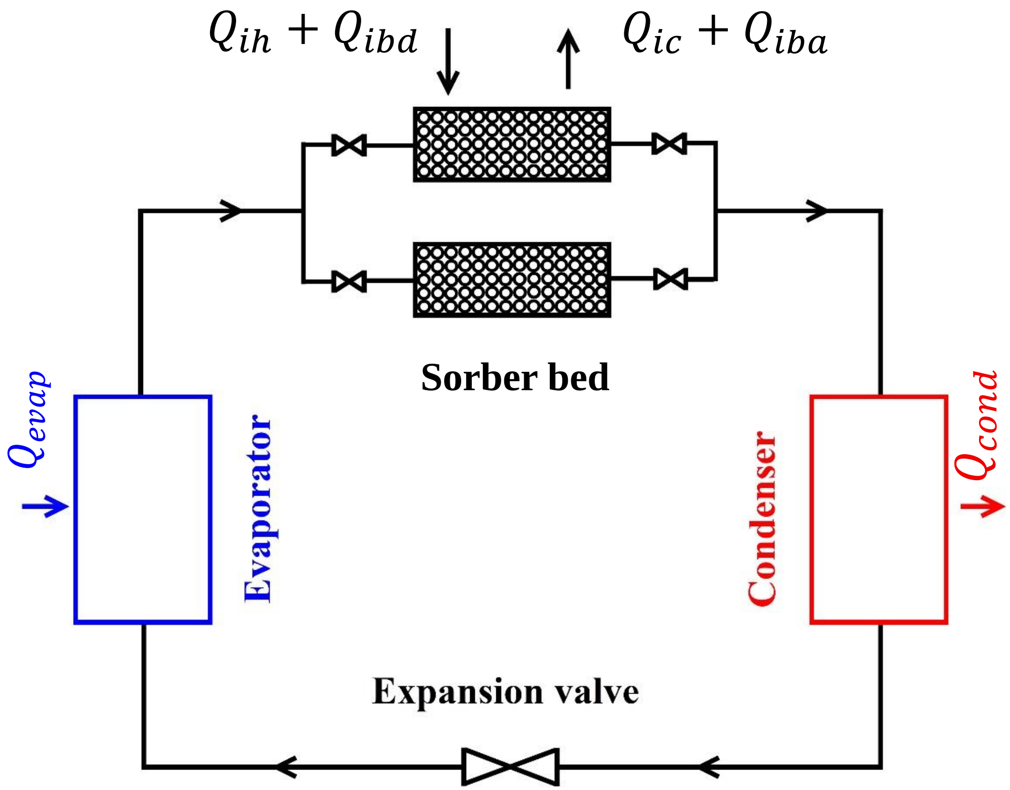
Sorption cooling systems

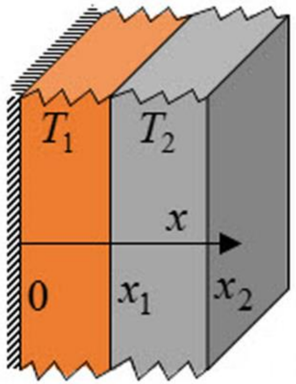


Dimensionless energy equation

$$\eta = \frac{x}{x_1}$$ $$Fo = \frac{\alpha_2 t}{x_1^{\,2}}$$ $$\theta = \frac{T_i(\eta, Fo) - T_{fin}(Fo)}{T_0}$$ $$\mu_i = \frac{\alpha_i}{\alpha_2}$$ $$\kappa_{i+1} = \frac{k_{i+1}}{k_i}$$ $$R_c = \frac{k_i A \cdot TCR}{x_1}$$

$$\mu_i \frac{\partial^2 \theta_i(\eta, Fo)}{\partial \eta^2} = \frac{\partial \theta_i(\eta, Fo)}{\partial Fo} + \frac{1}{T_0}\frac{dT_{fin}(Fo)}{dFo}$$

$$\theta_i(\eta, 0) = 0$$



$\eta = 0$

$$\frac{\partial \theta_1(0, Fo)}{\partial \eta} = 0$$

$\eta = 1$

$$\frac{\partial \theta_1(1, Fo)}{\partial \eta} = \kappa_2 \frac{\partial \theta_2(1, Fo)}{\partial \eta}$$

$$-\frac{\partial \theta_1(1, Fo)}{\partial \eta} = \frac{1}{R_c}\left(\theta_1(1, Fo) - \theta_2(1, Fo)\right)$$

$\eta = x_2 / x_1$

$$\theta_2(x_2 / x_1, Fo) = 0$$

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Orthogonal expansion technique

$$\theta_i(\eta, Fo) = \sum_{n=1}^{\infty} X_{in}(\eta)\Gamma_n(Fo)$$

Eigen-value Problem

$$\mu_i \frac{d^2 X_{in}(\eta)}{d\eta^2} + \beta_n{}^2 X_{in}(\eta) = 0$$

$$\frac{dX_{1n}(0)}{d\eta} = 0$$

$$\frac{dX_{1n}(1)}{d\eta} = \kappa_2 \frac{dX_{2n}(1)}{d\eta}$$

$$-\frac{dX_{1n}(1)}{d\eta} = \frac{1}{R_c}\left(X_{1n}(1) - X_{2n}(1)\right)$$

$$X_{2n}(x_2/x_1) = 0$$

Transcendental equation

$$\frac{1}{R_c\sqrt{\mu_1}}\tan(\frac{\beta_n}{\sqrt{\mu_2}})\tan(\frac{\beta_n}{\sqrt{\mu_1}}) - \frac{\kappa_1\beta_n}{\sqrt{\mu_1}\sqrt{\mu_2}}\tan(\frac{\beta_n}{\sqrt{\mu_1}}) + \frac{\kappa_1}{R_c\sqrt{\mu_2}} - \frac{1}{R_c\sqrt{\mu_1}}\tan(\frac{\beta_n x_2}{\sqrt{\mu_2}x_1})\tan(\frac{\beta_n}{\sqrt{\mu_1}})$$
$$-\frac{\kappa_1\beta_n}{\sqrt{\mu_1}\sqrt{\mu_2}}\tan(\frac{\beta_n x_2}{\sqrt{\mu_2}x_1})\tan(\frac{\beta_n}{\sqrt{\mu_2}})\tan(\frac{\beta_n}{\sqrt{\mu_1}}) + \frac{\kappa_1}{R_c\sqrt{\mu_2}}\tan(\frac{\beta_n x_2}{\sqrt{\mu_2}x_1})\tan(\frac{\beta_n}{\sqrt{\mu_2}}) = 0$$

Eigen-functions

$$X_{1n}(\eta) = \cos(\frac{\beta_n}{\sqrt{\mu_1}}\eta)$$

$$X_{2n}(\eta) = C_{2n}\cos(\frac{\beta_n}{\sqrt{\mu_2}}\eta) + D_{2n}\sin(\frac{\beta_n}{\sqrt{\mu_2}}\eta)$$

$$D_{2n} = \frac{\sqrt{\mu_2}\sin(\frac{\beta_n}{\sqrt{\mu_1}})}{-\kappa_1\sqrt{\mu_1}\left(\sin(\frac{\beta_n}{\sqrt{\mu_2}})\tan(\frac{\beta_n x_2}{\sqrt{\mu_2}x_1}) + \cos(\frac{\beta_n}{\sqrt{\mu_2}})\right)}$$

$$C_{2n} = -D_{2n}\tan(\frac{\beta_n x_2}{\sqrt{\mu_2}x_1})$$

M. Necati Özışık, Boundary value problems of heat conduction, International Textbook Company, 1968

Orthogonal expansion technique

$$\mu_i \frac{\partial^2 \theta_i(\eta, Fo)}{\partial \eta^2} = \frac{\partial \theta_i(\eta, Fo)}{\partial Fo} + \frac{1}{T_0}\frac{dT_{fin}(Fo)}{dFo}$$

Expansion of functions in the form:

$$f_i(\eta) = \sum_{n=1}^{\infty} f_n^*(Fo) X_{in}(\eta)$$

Orthogonal property of eigen-functions

$$\sum_{i=1}^{m} \frac{\kappa_i}{\mu_i} \int_{x_i}^{x} X_{in}(\eta) X_{in'}(\eta) d\eta = \begin{cases} 0 & for\ n \neq n' \\ const. & for\ n = n' \end{cases}$$

⬇

$$f_n^*(Fo) = \frac{1}{N} \sum_{i=1}^{m} \frac{\kappa_i}{\mu_i} \int_{x_i}^{x_{i+1}} f_i(\eta) X_{in}(\eta) d\eta$$

⬇

$$\frac{d\Gamma_n(Fo)}{dFo} + \beta_n^2 \Gamma_n(Fo) = \frac{1}{T_0} I_n^* \frac{dT_{fin}(Fo)}{dFo}$$

$$\Gamma_n(0) = h_n^*$$

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Temperature distribution inside a sorber bed

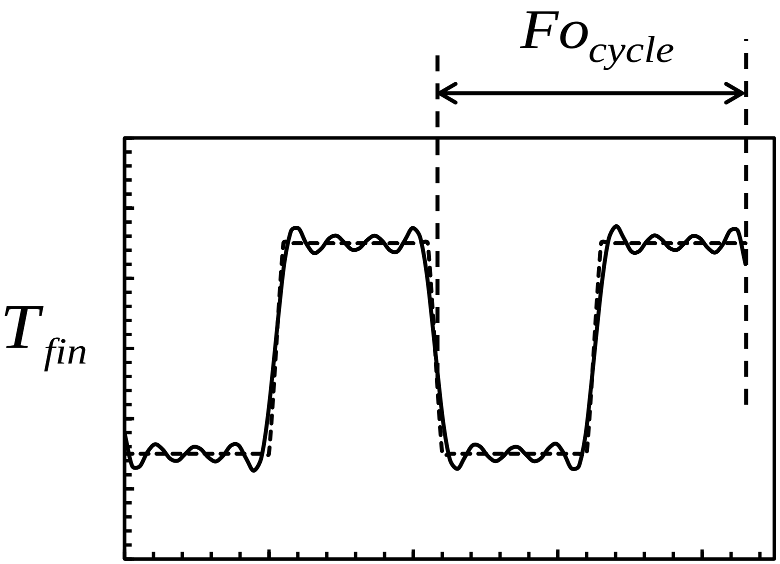
$$\theta_{sorb}(\eta, Fo) = \sum_{n=1}^{\infty} \cos\left(\frac{\beta_n}{\sqrt{\mu_1}}\eta\right) \left[f_n^* e^{-\beta_n^2 Fo} + \sum_{j=1}^{4} \left\{ r_{j,n} \left(\cos(j\omega Fo) - e^{-\beta_n^2 Fo} \right) + s_{j,n} \sin(j\omega Fo) \right\} \right]$$

$$\theta_{fin}(\eta, Fo) = \sum_{n=1}^{\infty} \left(C_{2n} \cos\left(\frac{\beta_n}{\sqrt{\mu_2}}\eta\right) + D_{2n} \sin\left(\frac{\beta_n}{\sqrt{\mu_2}}\eta\right) \right) \left[f_n^* e^{-\beta_n^2 Fo} + \sum_{j=1}^{4} \left\{ r_{j,n} \left(\cos(j\omega Fo) - e^{-\beta_n^2 Fo} \right) + s_{j,n} \sin(j\omega Fo) \right\} \right]$$

$r_{j,n}$ *and* $s_{j,n}$ *obtained from*

$$T_{HEX}(Fo) = a'_0 + \sum_{j=1}^{4} \left(a'_j \cos(j\Omega Fo) + b'_j \sin(j\Omega Fo) \right)$$

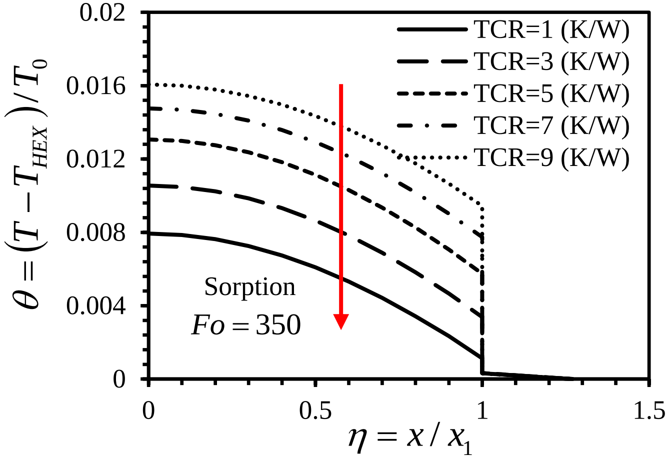
$$\Omega = \frac{2\pi}{Fo_{cycle}}$$



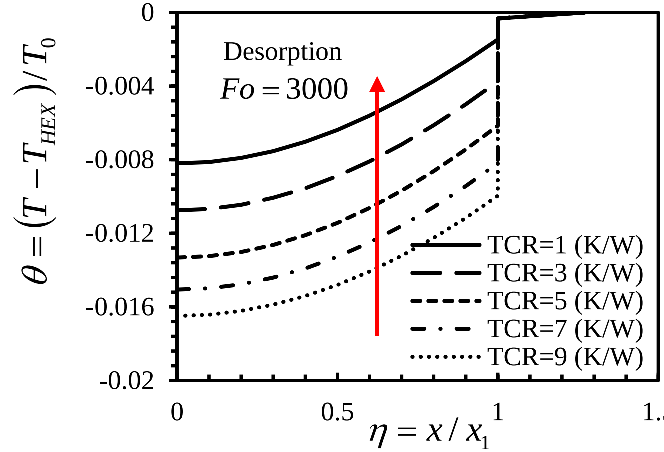
$$Fo = \frac{\alpha_2 t}{x_1^2}$$

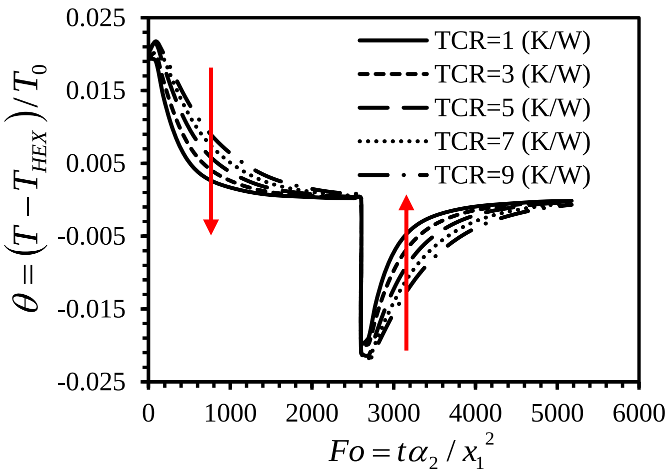
Effect of thermal contact resistance (TCR) on SCS performance

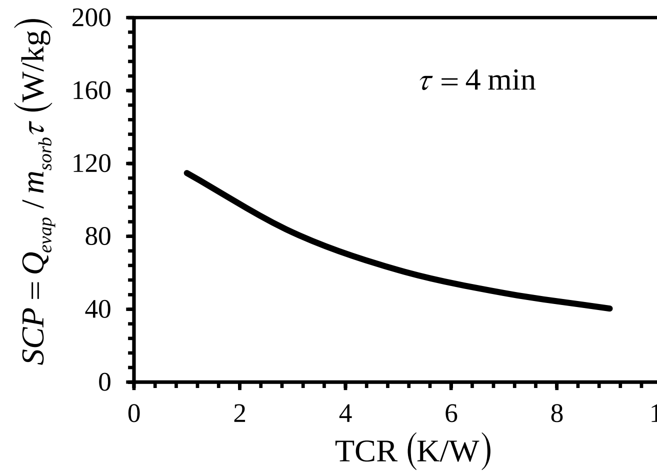
Sorption



Desorption







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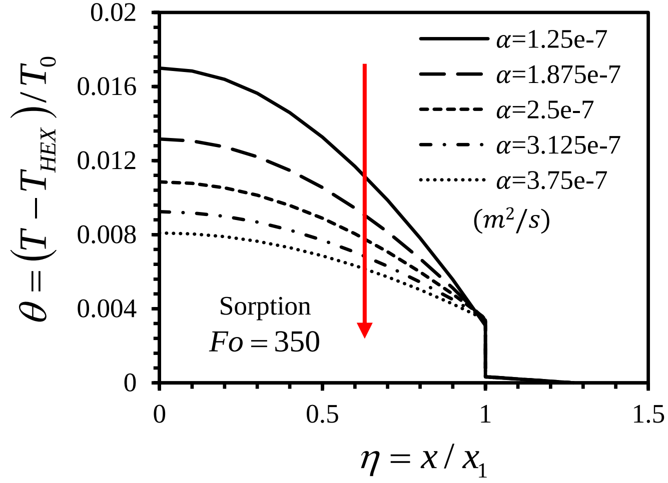
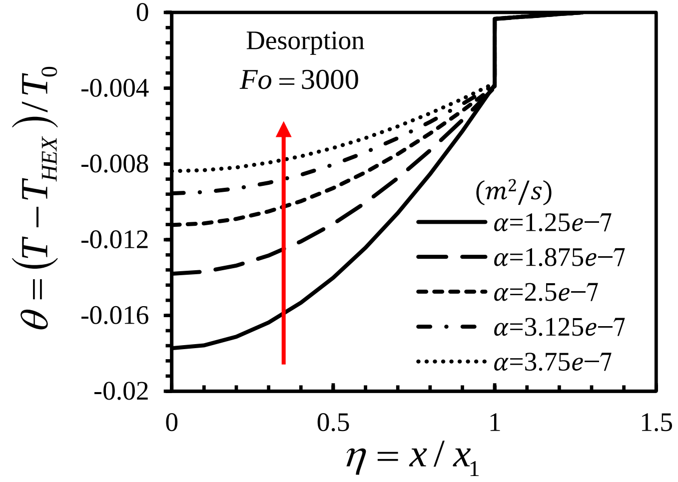
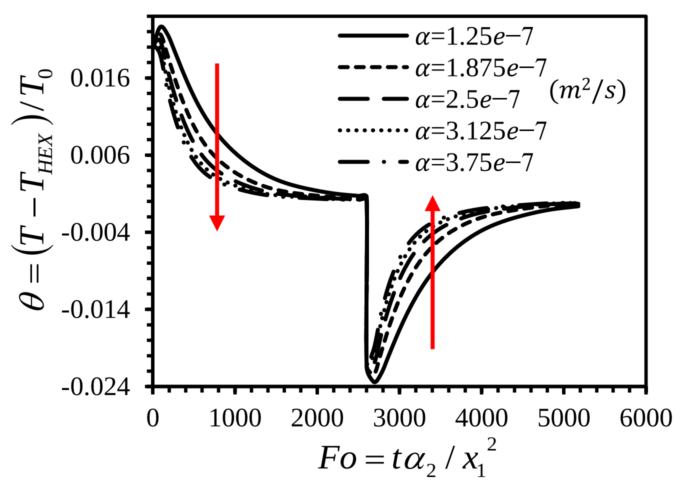
Effect of sorbent thermal diffusivity on SCS performance

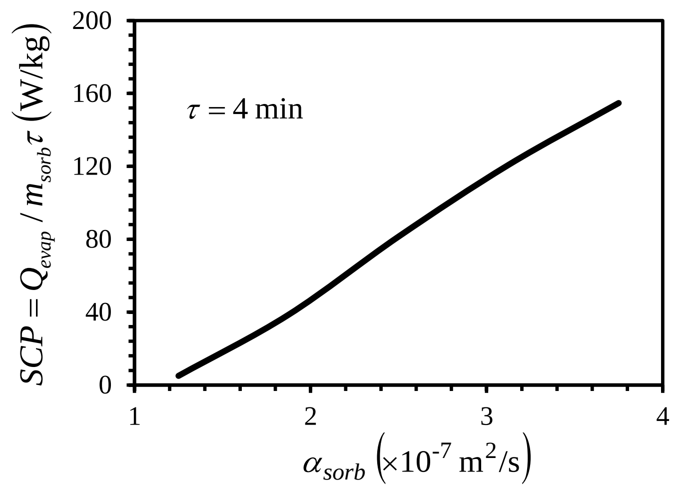
Sorption



Desorption







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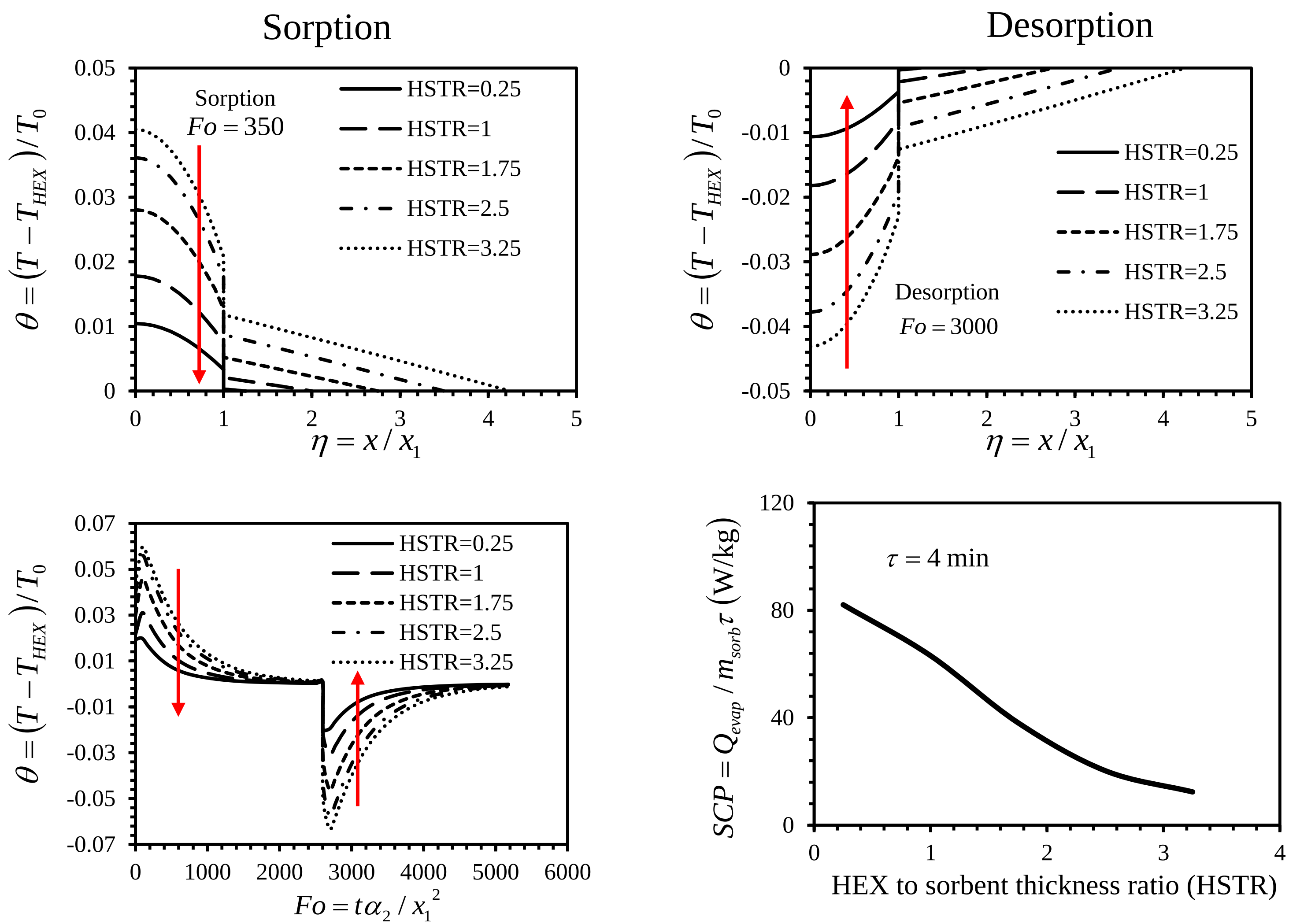
SFU Effect of HEX to sorbent thickness ratio (HSTR) on SCS performance



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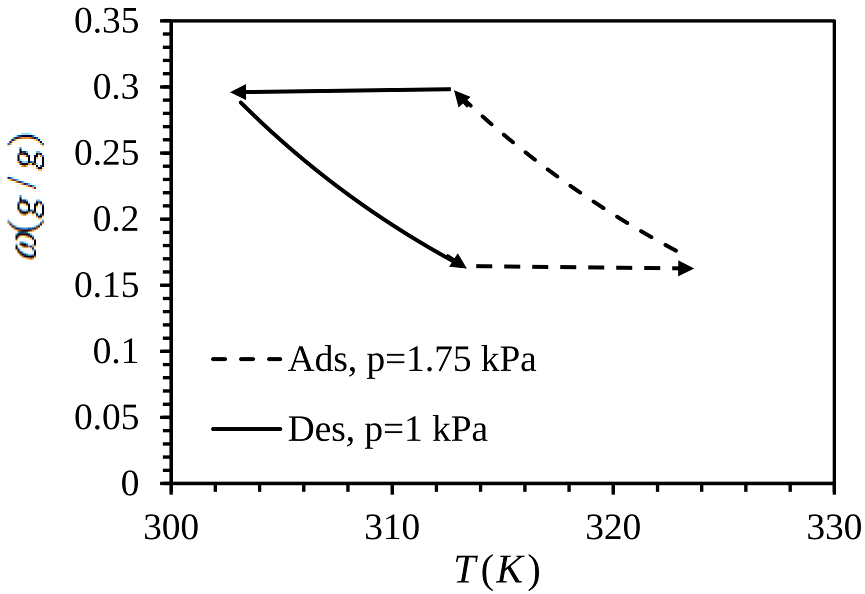
Scale analysis of energy equation



Uptake remains almost const.

$$\rho c\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2}+\rho H_{sorp}\frac{d\omega}{dt}\overset{Very small \Delta t}{\Rightarrow}\rho c\frac{\Delta T}{\Delta t}\sim\rho H_{sorp}\frac{\Delta\omega}{\Delta t}\Rightarrow\Delta\omega\sim\frac{c\Delta T}{H_{sorp}}=\mathrm{Ja}_{sorp}<0.005$$

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